525 Rec'd PCT/PTO 13 DEC 2000

FORM PTO-1390 (REV 10-94) U S DEPARTMENT OF COMMERCE PATENT AND TRADEMARK OFFICE		ATTORNEY'S DOCKET NUMBER
TRANSMITTAL LETTER TO THE UNITED STATES DESIGNATED/ELECTED OFFICE (DO/EO/US) CONCERNING A FILING UNDER 35 U.S.C. 371		9320.114USWO
		U S APPLICATION NO (If known, see 37 C F R 1.5) Unknown 09/719498
INTERNATIONAL APPLICATION NO. PCT/FR99/01902	INTERNATIONAL FILING DATE July 30, 1999	PRIORITY DATE CLAIMED July 30, 1998

TITLE OF INVENTION

METHOD FOR THE MAKING OF DIGITAL NYQUIST FILTERS WITH NULL INTERSYMBOL INTERFERENCE AND CORRESPONDING FILTERING DEVICE

APPLICANT(S) FOR DO/EO/US

SIOHAN et al.

Applicant herewith submits to the United States Designated/Elected Office (DO/EO/US) the following items and other information:

1. [X] This is a **FIRST** submission of items concerning a filing under 35 U.S.C. 371.
2. [ ] This is a **SECOND** or **SUBSEQUENT** submission of items concerning a filing under 35 U.S.C. 371.
3. [X] This express request to begin national examination procedures (35 U.S.C. 371(f)) at any time rather than delay examination until the expiration of the applicable time limit set in 35 U.S.C. 371(b) and PCT Articles 22 and 39(l).
4. [X] A proper Demand for International Preliminary Examination was made by the 19th month from the earliest claimed priority date.
5. [X] A copy of the International Application as filed (35 U.S.C. 371(c)(2))
   a. [X] is transmitted herewith (required only if not transmitted by the International Bureau).
   b. [X] has been transmitted by the International Bureau.
   c. [ ] is not required, as the application was filed in the United States Receiving Office (RO/US)
6. [X] A translation of the International Application into English (35 U.S.C. 371(c)(2)).
7. [X] Amendments to the claims of the International Application under PCT Article 19 (35 U.S.C. 371(c)(3))
   a. [ ] are transmitted herewith (required only if not transmitted by the International Bureau).
   b. [ ] have been transmitted by the International Bureau.
   c. [ ] have not been made; however, the time limit for making such amendments has NOT expired.
   d. [ ] have not been made and will not be made.
8. [ ] A translation of the amendments to the claims under PCT Article 19 (35 U.S.C. 371(c)(3)).
9. [X] An unsigned oath or declaration of the inventor(s) (35 U.S.C. 371 (c)(4)).
10. [ ] A translation of the annexes to the International Preliminary Examination Report under PCT Article 36 (35 U.S.C. 371(c)(5)).

Items 11. to 16. below concern document(s) or information included:

11. [X] An Information Disclosure Statement under 37 CFR 1.97 and 1.98.
12. [ ] An assignment document for recording. A separate cover sheet in compliance with 37 CFR 3.28 and 3.31 is included.
13. [X] A FIRST preliminary amendment.
    [ ] A SECOND of SUBSEQUENT preliminary amendment.
14. [ ] A substitute specification.
15. [ ] A change of power of attorney and/or address letter.
16. [X] Other items or information: Form 1449, 1 cited reference; Front page of PCT appln; International Preliminary Examination Report; International Search Report

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APPLICATION NO. (If known, see 37 CFR 1.5)	INTERNATIONAL APPLICATION NO	ATTORNEY'S DOCKET NUMBER
Unknown 09/719498	PCT/JP99/01902	9320.114USWO

17. [X] The following fees are submitted:

BASIC NATIONAL FEE (37 CFR 1.492(a) (1)-(5)):

Search Report has been prepared by the EPO or JPO **$860.00**

International preliminary examination fee paid to U.S. Patent and Trademark Office (37 CFR 1.492(a)(1)) **$690.00**

No international preliminary examination fee paid to USPTO (37 CFR 1.482) but international search fee paid to USPTO (37 CFR 1.445(a)(2)) $710.00

Neither international preliminary examination fee (37 CFR 1.482) nor international search fee (37 CFR 1.445(a)(3)) paid to USPTO **$1000.00**

International preliminary examination fee paid to USPTO (37 CFR 1.482) and all claims satisfied provisions of PCT Article 33(2)-(4) **$100.00**

				CALCULATIONS	PTO USE ONLY
ENTER APPROPRIATE BASIC FEE AMOUNT =				$860.00	
Surcharge of **$130.00** for furnishing the oath or declaration later than [ ] 20 [ ] 30 months from the earliest claimed priority date (37 CFR 1.492(e)).				$0	
CLAIMS	NUMBER FILED	NUMBER EXTRA	RATE		
Total claims	9 -20 =	0	X **$18.00**	**$0**	
Independent claims	2 -3 =	0	X **$80.00**	**$0**	
MULTIPLE DEPENDENT CLAIM(S) (if applicable)			+ **$270.00**	**$0**	
TOTAL OF ABOVE CALCULATIONS =				$860.00	
Reduction by 1/2 for filing by small entity, if applicable. Verified Small Entity Statement must also be filed (Note 37 CFR 1.9, 1.27, 1.28).				$0	
SUBTOTAL =				$860.00	
Processing fee of **$130.00** for furnishing the English translation later than [ ] 20 [ ] 30 months from the earliest claimed priority date (37 CFR 1.492(f). +				$0	
TOTAL NATIONAL FEE =				$860.00	
Fee for recording the enclosed assignment (37 CFR 1.21(h)). The assignment must be accompanied by an appropriate cover sheet (37 CFR 3.28, 3.31). $40.00 per property +				$0	
TOTAL FEES ENCLOSED =				$860.00	
				Amount to be: refunded	$0
				charged	$0

[X] Check(s) in the amount of $860.00 to cover the above fees is enclosed.

[ ] Please charge my Deposit Account No. ______________ in the amount of $ ____________ to cover the above fees. A duplicate copy of this sheet is enclosed.

[X] The Commissioner is hereby authorized to charge any additional fees which may be required, or credit any overpayment to Deposit Account No. 13-2725.

NOTE: Where an appropriate time limit under 37 CFR 1.494 or 1.495 has not been met, a petition to revive (37 CFR 1.137(a) or (b)) must be filed and granted to restore the application to pending status.

SEND ALL CORRESPONDENCE TO
John J. Gresens
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SIGNATURE John J. Gresens

NAME John J. Gresens

REGISTRATION NUMBER 33,112

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S/N Unknown PATENT

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

Applicant:	SIOHAN et al.	Examiner:	Unknown
Serial No.:	Unknown	Group Art Unit:	Unknown
Filed:	December 13, 2000	Docket No.:	9320.114USWO
Title:	METHOD FOR THE MAKING OF DIGITAL NYQUIST FILTERS WITH NULL INTERSYMBOL INTERFERENCE AND CORRESPONDING FILTERING DEVICE		

CERTIFICATE UNDER 37 CFR 1.10
'Express Mail' mailing label number: EL658339200US
Date of Deposit: December 13, 2000
I hereby certify that this correspondence is being deposited with the United States Postal Service 'Express Mail Post Office To Addressee' service under 37 CFR 1.10 on the date indicated above and is addressed to the Assistant Commissioner for Patents, Washington, D.C . 20231.
By: Chatia Lambert
Name: Chatia Lambert

PRELIMINARY AMENDMENT

Box PCT
Assistant Commissioner for Patents
Washington, D. C. 20231

Dear Sir:

In connection with the above-identified application filed herewith, please enter the following preliminary amendment:

IN THE SPECIFICATION

A courtesy copy of the present specification is enclosed herewith. However, the World Intellectual Property Office (WIPO) copy should be relied upon if it is already in the U.S. Patent Office.

IN THE CLAIMS

3. (Amended) Method according to claim 1, characterized in that, in said sending filter (12), a filtering step followed by a step of interpolation by a factor of M=4 is performed.

4. (Amended) Method according to claim 1, characterized in that, in said reception filter (15), a step of decimation by a factor M=4 is perfromed, followed by a filtering step.

5. (Amended) Method according to claim 1, characterized in that said sending filter (12) and/or said reception filter (15) have a structure in the form of at least one lattice.

REMARKS

The above preliminary amendment is made to remove multiple dependencies from claims 3, 4 and 5.

Applicants respectfully request that the preliminary amendment described herein be entered into the record prior to calculation of the filing fee and prior to examination and consideration of the above-identified application.

If a telephone conference would be helpful in resolving any issues concerning this communication, please contact Applicants' primary attorney-of record, John J. Gresens (Reg. No. 33,112), at (612) 371.5265.

Respectfully submitted,

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Dated: December 13, 2000

By [signature]
John J. Gresens
Reg. No. 33,112

JJG/tvm

-- CLAIM 1 --

$$F(z)=\left[F_0(z^4)\ F_1(z^4)\ \hat{F}_1(z^4)\ \hat{F}_0(z^4)\right]\begin{bmatrix}1\\ z^{-1}\\ z^{-2}\\ z^{-3}\end{bmatrix}$$

3. Method according to any of the claims 1 and 2 characterized in that, in said sending filter (12), a filtering step followed by a step of interpolation by a factor of M=4 is performed. -- CLAIM 1 --

4. Method according to any of the claims 1 à 3 characterized in that, in said reception filter (15), a step of decimation by a factor M=4 is performed, followed by a filtering step. -- CLAIM 1 --

5. Method according to any of the claims 1 to 4 characterized in that said sending filter (12) and/or said reception filter (15) have a structure in the form of at least one lattice.

6. Method according to claim 5, characterized in that said sending filter (12) and said reception filter (15) are each constituted by a pair of polyphase components respectively given by the following equations :

$$\begin{bmatrix}F_0\\ F_1\end{bmatrix}=gA(\alpha_n)\Lambda(z)A(\alpha_{n-1})\ldots\Lambda(z)A(\alpha_0)\begin{bmatrix}1\\ 0\end{bmatrix}$$

$$\begin{bmatrix}-\hat{F}_1\\ \hat{F}_0\end{bmatrix}=gA(\alpha_n)\Lambda(z)A(\alpha_{n-1})\ldots\Lambda(z)A(\alpha_0)\begin{bmatrix}0\\ 1\end{bmatrix}$$

with :

$$A(\alpha)=\begin{bmatrix}1 & \alpha\\ -\alpha & 1\end{bmatrix}\qquad \text{and}\qquad \Lambda(z)=\begin{bmatrix}1 & 0\\ 0 & z^{-1}\end{bmatrix}$$

where g is a non-null constant of standardisation and α_i are real coefficients.

7. Method according to claim 6, characterized in that it implements a two-lattice structure.

8. Method according to claim 6, characterized in that it implement a single-lattice structure working at a double frequency.

9. Device for the filtering of Nyquist digital signals with null inter-symbol interference designed to process a physical signal transmitted between a sender and a receiver through a transmission channel,

based on an Nth order $P(z) = F^2(z)$ symmetrical filter implementing an oversampling factor $M = 4$ and forming a matched pair comprising a sending filter (12) and a reception filter (15),

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Method for the making of digital Nyquist filters with null inter-symbol interference and corresponding filtering device

The field of the invention is that of filtering of digital signals. More specifically, the invention relates to the making of Nyquist filters such as those used for example in baseband and single-carrier transmission.

Nyquist filters play an essential role in transmission systems. In most cases, this filtering is distributed between sending and reception in the form of two filters known as Nyquist root filters. When sending, the aim especially is to limit the band of the signal sent. At reception, the filter must eliminate all the different noises that may damage the useful signal, especially the interference noise due to adjacent channels. In modern systems, these filters are made in digital form. There are many methods to synthesize these filters. A key element of this synthesis, apart from compliance with frequency specifications, is the need to obtain the smallest possible inter-symbol interference (ISI).

To ensure the most efficient possible conditions of transmission, it is sought generally to obtain real coefficient filters which, ideally, will meet the following criteria:

- null inter-symbol interference;
- frequency selectivity;
- sending and reception filters forming matched pairs;
- phase linearity.

Appendix 1 describes these different aspects and details and the corresponding constraints. This appendix, just like the following ones, is of course an integral part of the present description.

A complete transmission system must generally also carry out other indispensable functions such as:

- analog filtering at sending which is designed to reject the output harmonics from the digital filter as well as analog filtering at reception whose function is to limit the spectrum of the signal to be sampled solely to the useful band;
- digital-analog conversions (DAC) at sending, and analog-digital conversions (ADC) at reception.

The invention essentially relates to the obtaining of digital filters, and no detailed description shall be given of the functions relating to the analog part. It must be noted however that, in practice, the analog filtering to be implemented is substantially less restrictive in terms of steepness of the filters than digital filtering. More specifically, the complexity of the embodiment of the rejection filter at sending is directly related to the value of the oversampling factor chosen.

As for the converters, the most notable fact from the viewpoint of filtering is that of digital-analog conversion which introduces a sine(x)/x filtering that may be conventionally compensated for by the analog filtering. This processing can also be done digitally.

Furthermore, for reasons of implementation, the filters are generally sampled at double or quadruple frequency.

Figure 1 illustrates the digital part of a communications system of this kind. The signal to be sent X(z) 11 supplies the sender 12 which delivers a filtered signal Y(z) 13 transmitted through a transmission channel 14 to a receiver 15 that delivers the output signal S(z) 16. The signal X(z) 11 is first of all subjected to oversampling 121 and then to a filtering 122 $F_t(z)$. In the receiver, the signal undergoes a reception filtering $F_r(z)$ 151 and then a decimation 152. With T as the elementary delay linked to the sending and reception digital filters, we have $T_S = MT$, where T_S is the symbol duration and M the factor of oversampling.

The value of having a high oversampling factor is especially the fact that it facilitates the making of the analog filter that will follow. One drawback is that it correspondingly increases the speed of the converter (DAC). In practice, the most reasonable compromise corresponds to a value of M = 2 or 4. The invention relates specifically to the case where the oversampling factor is M = 4.

Nyquist filter construction techniques are already known. However, the different classes of known filters always entail the release of one of the following desired properties : zero inter-symbol interference, phase linearity (at least at sending) and the pair of matched filters.

Often, in communication systems, it becomes necessary to accept that the filtering introduces a non-zero ISI. This however sometimes raises problems, for

example in the case of modulations with very large numbers of states (64, 256, ...).

It is an object of the invention especially to overcome these drawbacks of the prior art.

More specifically, it is an object of the invention to provide a method for the making of Nyquist filters with null inter-symbol interference that also meets the desired conditions of frequency selectivity and phase linearity in a matched pair configuration.

Another aim of the invention is to provide a method such as this that enables easy practical implementation of the filters obtained and takes account of certain practical constraints such as the effects of the quantification of the coefficients. More specifically, a goal of the invention is to provide a method to ensure the maintenance of the property of cancellation of inter-symbol interference after this quantification.

Another aim of the invention is to provide a method of this kind that can be implemented by means of a cascade structure, especially in lattice form.

These aims as well as others that shall appear hereinafter are achieved according to the invention by means of a method for the making of a digital Nyquist filter with null inter-symbol interference designed to process a physical signal transmitted between a sender and a receiver through a transmission channel. This filter is an Nth order $P(z) = F^2(z)$ symmetrical filter implementing an oversampling factor M=4 and forming a matched pair comprising a sending filter and a reception filter whose polyphase breakdown of F(z) can be written as follows:

$$F(z) = F_0(z^4) + z^{-1}F_1(z^4) + z^{-2}F_2(z^4) + z^{-3}F_3(z^4).$$

According to the invention, N is chosen to be different from 4n, n being an integer and the coefficients of the polyphase breakdown of F(z) are such that:

- If N=4n+1, $F_1(z)\hat{F}_1(z) + z^{-1}F_2(z)\hat{F}_2(z) = \gamma z^{-n}$
- If N=4n+2, $2F_0(z)\hat{F}_0(z) + F_1^2(z) + z^{-1}F_3^2(z) = \gamma z^{-n}$
- If N=4n+3, $F_0(z)\hat{F}_0(z) + F_1(z)\hat{F}_1(z) = \gamma z^{-n}$

$\hat{F}$ being the mirror symmetry of F and γ being a non-null constant.

Thus, the method of the invention, by construction, ensures that the ISI is perfectly null.

Preferably, N is equal to 4n+3 or 4n+1 and:

said sending filter performs an interpolation by a factor M = 4 and has a circuit arrangement corresponding to a polyphase breakdown known as the type II breakdown, such that:

$$F(z)=\begin{bmatrix} z^{-3} & z^{-2} & z^{-1} & 1 \end{bmatrix}\begin{bmatrix} \hat{F}_0(z^4) \\ \hat{F}_1(z^4) \\ F_1(z^4) \\ F_0(z^4) \end{bmatrix}$$

and said reception filter performs a decimation by a factor M = 4 and has a circuit arrangement corresponding to a polyphase breakdown known as the type I breakdown, such that:

$$F(z)=\begin{bmatrix} F_0(z^4) & F_1(z^4) & \hat{F}_1(z^4) & \hat{F}_0(z^4) \end{bmatrix}\begin{bmatrix} 1 \\ z^{-1} \\ z^{-2} \\ z^{-3} \end{bmatrix}$$

The filters thus obtained are simple to make and implement.

Advantageously, in said sending filter, there is performed a filtering step followed by a step of interpolation by a factor M = 4. Similarly, in said reception filter, advantageously a step of decimation by a factor M = 4 is performed, followed by a filtering step.

This construction (permutated structure) reduces the rate of the operations by a factor 4.

According to a preferred embodiment of the invention, said sending filter and/or said reception filter has a structure in the form of at least one lattice.

Indeed, in this form of circuit arrangement, the constraint of perfect reconstruction is structurally integrated.

Advantageously, said sending filter and said reception filter are each constituted by a pair of polyphase components respectively given by the following equations:

$$\begin{bmatrix} F_0 \\ F_1 \end{bmatrix} = gA(\alpha_n)\Lambda(z)A(\alpha_{n-1})\ldots\Lambda(z)A(\alpha_0)\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -\hat{F}_1 \\ \hat{F}_0 \end{bmatrix} = gA(\alpha_n)\Lambda(z)A(\alpha_{n-1})\ldots\Lambda(z)A(\alpha_0)\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

with : $$A(\alpha) = \begin{bmatrix} 1 & \alpha \\ -\alpha & 1 \end{bmatrix} \quad \text{et} \quad \Lambda(z) = \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix}$$

where g is a non-null constant of standardization and α_i are real coefficients.

According to one particular embodiment of the invention, the method implements a two-lattice structure. According to another embodiment, it may implement a single-lattice structure working at a double frequency.

The invention also relates of course to filtering devices obtained by means of the above-described method. These are therefore devices for the filtering of Nyquist digital signals with null inter-symbol interference designed to process a physical signal transmitted between a sender and a receiver through a transmission channel, based on an Nth order $P(z) = F^2(z)$ symmetrical filter implementing an oversampling factor $M = 4$ and forming a matched pair comprising a sending filter and a reception filter, the polyphase breakdown of F(z) of this symmetrical filter being written as follows:

$$F(z) = F_0(z^4) + z^{-1}F_1(z^4) + z^{-2}F_2(z^4) + z^{-3}F_3(z^4).$$

According to the invention, N is different from 4n, n being an integer, and:

- If N=4n+1, $F_1(z)\hat{F}_1(z) + z^{-1}F_2(z)\hat{F}_2(z) = \gamma z^{-n}$
- If N=4n+2, $2F_0(z)\hat{F}_0(z) + F_1^2(z) + z^{-1}F_3^2(z) = \gamma z^{-n}$
- If N=4n+3, $F_0(z)\hat{F}_0(z) + F_1(z)\hat{F}_1(z) = \gamma z^{-n}$

$\hat{F}$ being the mirror symmetry of F and γ being a non-null constant.

Other features and advantages of the invention shall appear more clearly from the following description of a preferred embodiment given by way of a simple, illustrative and non-restrictive example and from the appended drawings, of which:

- Figure 1, already commented upon in the introduction, provides a diagrammatic illustration of the digital part of a communications system;
- Figure 2, commented on in Appendix 1, represents a typical frequency specification of a sending filter;
- Figure 3, commented upon in Appendix 2, gives a general view of a bank of filters with two sub-bands;
- Figure 4 illustrates an embodiment of the analysis part of a bank of para-unitary filters with two sub-bands of Figure 3 in lattice form:
- Figure 5 recalls the structure of a sending/reception filtering system in the linear and para-unity case commented on hereinafter;
- Figures 6a and 6b give a view, for a 4n+3 order filter of two forms of circuit arrangement of the sending filter of Figure 5, using a polyphase breakdown respectively in a direct structure and a permutated structure;
- Figures 7a and 7b provide an illustration, in the case of the 4n+3 order, of the reception filter of Figure 5 also in the context of polyphase breakdown respectively according to a direct structure and a permutated structure;
- Figures 8a and 8b show a lattice block of the sending/reception filters of Figures 6a, 6b, 7a and 7b respectively in the form of a direct lattice and a lattice with inverted outputs;
- Figure 9 gives a detailed view of a 4n + 3 order Nyquist root sending filter with two lattices implementing the inverted lattices of Figure 8b;
- Figure 10 shows a 4n+3 order Nyquist reception filter implementing four lattices;
- Figure 10b shows a reception filter implementing a direct lattice according to Figure 8a (see Appendix 4);
- Figure 11 recalls the equations used in the case of the inverted lattice of Figure 9;

- Figure 12 shows another embodiment of the sending filter implementing a single lattice and working at a double rate;
- Figure 13 shows an exemplary response in optimized frequencies of a 43rd order Nyquist root filter according to the invention;
- Figure 14 illustrates the embodiment of a block Z in the case of a 4n+2 order filter as discussed in Appendix 3;
- Figure 15 shows the circuit arrangement block of the matrix P^T also in the context of the embodiment of Appendix 3;
- Figure 16 shows the full circuit arrangement diagram of a 4-input and 4-output block for the 4n+2 order referenced Ma and discussed in Appendix 3;
- Figure 17 shows an embodiment of a half Nyquist sending filter for the 4n+2 order;
- Figures 18 and 19 (block N of Figure 18) illustrate a half Nyquist reception embodiment for the 4n+2 order discussed in Appendix 3;
- Figures 20 and 21 respectively show the sending and reception Nyquist root filters for the 4n+1 order (or 4n+5 order) as discussed in Appendix 5;
- Figures 22 and 23 illustrate the results obtained by means of the mode of generic synthesis commented upon in Appendix 6.

As mentioned already, the method of the invention can be used to make filters with M = 4 verifying the totality of the following criteria:

- ISI null by construction;
- linear phase filters;
- sending and reception filters forming a matched pair.

Appendix 2, after developing the analogy between a Nyquist pair with M = 4 and a bank of filters orthogonal to two sub-bands; shows that the two situations may occur according to the order of the filter. More specifically, it is shown that there is no solution for the N = 4n order filters and that the cases N = 4n + 1 and N = 4n + 3 are favored (theorem 2) in that they produce similar polyphase components. The condition for obtaining null inter-symbol interference is specified in the theorem 1.

The case of the 4n + 3 filters is described hereinafter. It proves to be the case used to obtain the simplest embodiment. The case 4n + 2 is processed in Appendix 3 and the case 4n + 1 (or 4n + 5) in Appendix 5.

As a rule, the basic diagram implemented in shown in Figure 5. As compared with the initial general diagram of Figure 1, it may be noted that $F_T(z)$ 122 and $F_R(z)$ 151 are replaced by F(z). This is indeed a linear phase matched pair.

We must also note the introduction of a delay line z-r51 where r is a positive integer value dependent on the order, which in all cases makes it possible to obtain a null ISI with a given delay. The multiplier factor g 52 is used to ensure that, except for the delay, the output is quite identical to the input.

In the case where N = 4n + 3, because of the relationships between the different polyphase components F(z), the expression (20) of Appendix 2 can be rewritten as follows:

$$F(z) = F_0(z^4) + z^{-1}F_1(z^4) + z^{-2}\hat{F}_1(z^4) + z^{-3}\hat{F}_0(z^4) \quad (31)$$

It is known that, apart from the delay, the sending and reception filters are identical. However, if it is sought to draw the best advantage of the multi-rate processing operations, namely interpolation at sending and decimation at reception, the structural drawings will not be exactly the same.

At sending, where an interpolation by a factor M = 4 is made, the forms of circuit arrangement shown in Figures 6a and 6b correspond to those known as type II polyphase breakdown [4] (the references cited are assembled in Appendix 7), given in the case of the invention by:

$$F(z) = \begin{bmatrix} z^{-3} & z^{-2} & z^{-1} & 1 \end{bmatrix} \begin{bmatrix} \hat{F}_0(z^4) \\ \hat{F}_1(z^4) \\ F_1(z^4) \\ F_0(z^4) \end{bmatrix} \quad (32)$$

In one of the depictions (Figure 6b), it is possible to note the permutation of the expansion (61) and filtering (62) operations used to carry out an equivalent processing by working at a rate four times lower than in the case of the initial diagram (Figure 6a).

In the reception part, the filtering is associated with a decimation operation for which a more appropriate writing of F(z) is that of what is called the type I [4] polyphase decomposition which in the present case can be written as follows:

$$F(z)=\left[F_0(z^4)\ F_1(z^4)\ \tilde{F}_1(z^4)\ \tilde{F}_0(z^4)\right]\begin{bmatrix}1\\ z^{-1}\\ z^{-2}\\ z^{-3}\end{bmatrix} \tag{33}$$

It will be noted that, with reference to Figure 5, the time limit is such that r = 1.

Two variants of the circuit arrangement of this equation are given in Figures 7a and 7b (symmetrically with Figures 6a and 6b).

The analogy between the 4n+3 order filter F(z) and the 2n +1 order bank of filters described by the theorem 3 (Appendix 2) is used also to rewrite the equation in the form:

$$\begin{bmatrix}F_0(z) & -\hat{F}_1(z)\\ F_1(z) & \hat{F}_0(z)\end{bmatrix}=gA(\alpha_n)\Lambda(z)A(\alpha_{n-1})\ldots\Lambda(z)A(\alpha_0) \tag{34}$$

The two pairs of polyphase components which are mirrors of each other are therefore deduced from each other by the following two equations:

$$\begin{bmatrix}F_0\\ F_1\end{bmatrix}=gA(\alpha_n)\Lambda(z)A(\alpha_{n-1})\ldots\Lambda(z)A(\alpha_0)\begin{bmatrix}1\\ 0\end{bmatrix} \tag{35}$$

and

$$\begin{bmatrix}-\tilde{F}_1\\ \tilde{F}_0\end{bmatrix}=gA(\alpha_n)\Lambda(z)A(\alpha_{n-1})\ldots\Lambda(z)A(\alpha_0)\begin{bmatrix}0\\ 1\end{bmatrix} \tag{36}$$

In each of these cases, the lattice structure shown in Figures 8a and 8b is the same.

The two variants, with direct or inverted outputs of Figure 8, are then used to set up the sending and reception filters according to the diagrams respectively shown in Figures 9 and 10. It will be noted that the symbols at output of the filters indicate only which are the active filters.

By using the polyphase decomposition of the input signal, it is then possible to determine an equivalent diagram with only one lattice which however, in this case, is constrained to work at the double frequency.

In the diagram proposed, the inverted lattice is presented, it being known of course that the direct lattice leads to a diagram of the same type. Figure 11 recalls the basic equations of this lattice.

The output signal of the sending filter is written as follows:

$$Y(z)=z^{-1}\left[F_0(z^4)+z^{-1}F_1(z^4)+z^{-2}\hat{F}_1(z^4)+z^{-3}\hat{F}_0(z^4)\right]X(z^4) \qquad (37),$$

which can be rewritten as follows:

$$\begin{aligned}Y(z)&=z^{-1}\left[F_0(z^4)+z^{-2}\hat{F}_1(z^4)\right]X(z^4)+z^{-2}\left[F_1(z^4)+z^{-2}\hat{F}_0(z^4)\right]X(z^4)\\&=\left[Y_0(z^2)+z^{-1}Y_1(z^2)\right]z^{-1}\end{aligned} \qquad (38)$$

A division by 2 of the expansion factor (↑ 4 ≡ ↑ 2 ↑ 2), and then a permutation of the expansion of 2 with the filtering produces filtered outputs such that:

$$\begin{aligned}Y_0(z)&=\left[F_0(z^2)X(z^2)+z^{-1}\hat{F}_1(z^2)X(z^2)\right]\\Y_1(z)&=\left[F_1(z^2)X(z^2)+z^{-1}\hat{F}_0(z^2)X(z^2)\right]\end{aligned} \qquad (39).$$

In $Y_1(z)$, it is possible to recognize the higher output of the inverted lattice working at double rate and, plus or minus the sign of the term as a factor of z^{-1}, in $Y_0(z)$ it is possible to recognize the lower output. To recover exactly the same expression, it is therefore enough to multiply this output of the inverted lattice by $(-1)^n$.

One embodiment based on this principle is shown in Figure 12.

For the reception part, it is possible to consider two cases:

- the general case, with four lattices, which will need all the desired properties (Figure 10);
- the simplified diagram for which only the null ISI and the phase linearity at sending are guaranteed (Figure 10b).

This particular aspect is discussed in Appendix 4.

In the case of Figure 9, the system works at the lowest frequency. The cost of each filter corresponds to a lattice structure with n+1 cells plus that of a sign

inversion. To obtain a unit gain, a multiplication by the factor g generally must be accounted for in the diagram of the reception filter.

The computations for each elementary filter correspond to the two multipliers and adders. However, as in the case of each lattice, one of the inputs is at zero, and a multiplier and an adder can therefore be eliminated. The cost is therefore one multiplier and one adder for this first cell and double for the following, namely (2n + 1) multipliers and (2n +1) adders. For each filter, we will therefore have (4n + 2) multipliers and (4n + 2) adders. At the lowest frequency, we will therefore have to make (4n + 2) MPU and (4n +2) APU.

For the reception part, in the case of the simplified diagram illustrated by Figure 10b, the operational complexity is that of a single lattice rather than that of three adders, the multiplier by g and the inverter. The computations performed at the lowest rate can be estimated at (2n + 3) MPU, (2n + 5) APU and one inversion.

In the general case illustrated by Figure 10, at the lowest frequency, the complexity is (8n + 4) MPU and APU and one inversion.

The method of synthesis is performed for example in three steps, illustrated here below by the computation of a 43rd order filter.

- Step 1: Using the synthesis tools [3], it is possible to obtain a first set of transversal coefficients that are close to null ISI. These are the initial coefficients of Table 1.
- Step 2: By identification, we can compute the corresponding initial lattice coefficients given in the Table 2. Since the initial set of transversal coefficients is not exactly at null ISI, a new set of transversal coefficients is thus obtained known as reconstructed coefficients (cf. Table 1).
- Step 3: A local optimizing of the initial lattice coefficients then provides for slight improvement in the frequency response of the filter. Tables 1 and 2 give the values of these lattice and transversal coefficients after this optimization (Tables 1 and 2).

Table 1 : Transversal coefficients of the 43rd degree symmetrical filters

i	Initial	Reconstructed	Optimizal
0	0.006302365568	0.006302365568	0.00374315552336009
1	-0.01335763186	-0.01335763186	-0.00764365688411938
2	-0.003586256877	0.000139948824594686	0.000256865997947587
3	0.003281590529	0.000296616382849522	0.00052452951560661
4	0.008254154585	0.00825445268157875	0.00210146572675043
5	0.008753151633	0.00875251982827294	0.00961384935212083
6	0.002954708412	-0.000818038274240153	-0.000565430770503378
7	-0.006948650815	-0.00231664836261536	-0.00210883853472157
8	-0.01474214438	-0.0147620230725521	-0.00994253905900314
9	-0.01369718742	-0.0136538137768085	-0.0139493173848354
10	-0.001682000235	0.00199238622180159	0.00140539009490696
11	0.01571460254	0.00939101589973353	0.00785652590927169
12	0.026870884	0.0269017853424573	0.0234276140713132
13	0.02112355083	0.0209715158594175	0.0216783222304717
14	-0.003340088297	-0.0032097877874163	-0.00339966101869637
15	-0.03511243314	-0.0308000965511264	-0.0274417538899884
16	-0.05300379917	-0.0530202725288858	-0.0511658629696837
17	-0.0364661812	-0.0360408764445379	-0.0377795925012697
18	0.0221698768	0.0140041435375816	0.0161657062857809
19	0.1105730906	0.116698398397514	0.109490464868349
20	0.1994492114	0.199658215352476	0.199471282748299
21	0.2550483644	0.253344105750997	0.257329314844394

Table 2 Coefficients of the lattice structures

	Initial	Optimizal
$\tilde{g}_0$	0.006302365568	0.003743155523
α_1	2.1194631945539	2.04203561311232
α_2	- 0.7583059650965	- 0.71854479175763
α_3	0.6811397085317	0.73802350460893
α_4	- 0.4637577935126	- 0.55360950120952
α_5	0.4470434662649	0.51415010432349
α_6	- 3.7431288142180	- 5.34790460147966
α_7	- 1.5196264288798	- 1.52189568702330
α_8	1.2422617481334	1.44910973460487
α_9	- 0.5612421724760	- 0.63596605846521
α_{10}	0.1588040486787	0.18869460160260
α_{11}	- 0.0222057611677	- 0.06862284945003

The frequency response for the 43rd-order filter computed in this way is shown in Figure 13.

This is a first result that may be further refined. Furthermore, of course, other computation techniques may be considered for different technical orders and constraints.

It will be recalled especially that the case $N = 4n + 2$ is presented in the Appendix 3. Furthermore, Appendix 5 discusses the case $N = 4n + 1$ (or $4n + 5$).

Details of the assessment of the complexity of the reception filters and a method of generic synthesis are developed in Appendix 6. It will be noted that this method of synthesis can be applied to all three types of approaches described here above.

APPENDIX 1

The distribution of the Nyquist filtering on two filters therefore commonly called Nyquist root filters or, in short, half-Nyquist filters, must take account of the constraints related to the ISI, the frequency specifications, the linearity of the phase and what is called the matched pair property. In this appendix, we define these different notions assuming, to simplify the notation, that $T = 1$. Furthermore, in the context of the embodiment, we shall deal only with the case of the finite pulse response filters (FIR).

1 - Intersymbol interference (ISI)

Let $x(n)$ be the input signal and $X(z)$ its z transform, namely $X(z) = \sum_n x(n)z^{-n}$ and let it be agreed that all the signals of the transmission system will be referenced according to the same principle. The null ISI constraint amounts to the dictating, when there is no noise, of the output signal referenced $S(z)$ which is identical, except for a delay, to $X(z)$.

Let $Y(z)$ be the output signal from the sender. It can be written as follows:

$$Y(z) = F_T(z)X(z^M). \tag{1}$$

At output of the reception filter, the signal referenced $U(z)$ has the following expression:

$$U(z) = F_T(z)F_R(z)X(z^M). \tag{2}$$

The output signal is therefore quite simply:

$$S(z) = \frac{1}{M}\sum_{k=0}^{M-1} U(z^{\frac{1}{M}}w^k) = \left[\frac{1}{M}\sum_{k=0}^{M-1} F_T(z^{\frac{1}{M}}w^k)F_R(z^{\frac{1}{M}}w^k)\right] X(z), \tag{3}$$

where $w = e^{\frac{2j\pi}{M}}$ is the Mth root of unity. The null ISI condition can then be summarized in the following equation:

$$\frac{1}{M}\sum_{k=0}^{M-1} F_T(z^{\frac{1}{M}}w^k)F_R(z^{\frac{1}{M}}w^k) = z^{-d}, \tag{4}$$

where d corresponds to the delay introduced by the two filters.

Let $P(z)$ be the filter produced, i.e. $P(z) = F_T(z)F_R(z)$. In a formal method commonly used in with multi-rate signal processing, the equation (4) can then be written as follows:

$$P(z)\downarrow_M = z^{-d}. \tag{5}$$

A filter $P(z)$ that verifies this relationship is a Nyquist filter or again an Mth band filter.

Let n_T and n_R be the respective orders of the sending and reception FIR filters. The filter P is therefore an $n_P = n_T + n_R$ order filter and can be put in the following form:

$$P(z) = \sum_{n=0}^{n_P} p(n) z^{-n}. \tag{6}$$

If the equation (5) is not verified, it is commonly the "distance" that is verified with respect to the property by either of the following expressions:

$$D_1 = \frac{1}{|\, p(d) \,|} \sum_{kM \neq d} |\, p(d - kM) \,|, \tag{7}$$

where

$$D_2 = \frac{1}{p^2(d)} \sum_{kM \neq d} p^2(d - kM). \tag{8}$$

2. The frequency specifications

Figure 2 shows a typical frequency specification for digital transmission filters:

- The roll-off factor defines, for these lowpass filters, the passband by

$[0, \omega_p = \frac{\pi}{M}(1 - \rho)]$ and the attenuated band by $[\omega_s = \frac{\pi}{M}(1 + \rho), \pi]$

- The passband ripple specifications referenced δ_1 and an attenuated band referenced δ_2 are generally the same for the sending and reception filters.

- At the pulsation π/M, generally the constraint $F_R(\pi/M) = F_T(\pi/M) = \sqrt{2}/2$ is dictated.

3 - The phase linearity

The choice of characteristic phase frequency filters that are perfectly linear is generally recommended for digital transmission systems [1, p. 325]. This especially has the advantage of preserving the instants of passage through zero of the transmitted binary trains.

4 - The matched pair of filters

In the case of linear modulation, if we assume that ISI is zero for the entire transmission system and that the channel noise is additive, white and Gaussian (BBAG), it is known that the matched pair, namely $F_T(z) = z^{-N} F_R(z^{-1})$ with $N = n_T = n_R$, the order of each filter, is optimal for the criterion of the signal-to-noise ratio (SNR) [2, pp. 51 sq.]. The SNR which is equal to $\frac{E}{N_0}\frac{1}{\|F_R\|^2\|F_T\|^2}$ [3] then reaches its maximum with $\|F_R\|^2\|F_T\|^2 = 1$.

APPENDIX 2

1 - Orthogonal filter banks with two sub-bands

A filter bank with two sub-bands can be shown according to the diagram of Figure 3. In order to meet the condition of non-aliasing, the synthesis filters $G_0(z)$ and $G_1(z)$ are directly deduced from the analysis filters by $G_0(z) = H_1(-z)$ and $G_1(z) = -H_0(-z)$.

It can then be shown [4] that the condition of perfect reconstruction (PR) can be expressed solely from the polyphase matrix $H_{[H_0,H_1]}$ (z) of the analysis bank:

$$\mathrm{H}_{[H_0,H_1]}(z) = \begin{bmatrix} H_{0,0}(z) & H_{0,1}(z) \\ H_{1,0}(z) & H_{1,1}(z) \end{bmatrix}, \qquad (9)$$

where the values of $H_{k,l}(z)$ are the polyphase components for each filter $H_k(z)$, namely:

$$H_k(z) = \sum_{l=0}^{1} z^{-l} H_{k,l}(z^2) \qquad (10)$$

In the case of the orthogonal banks with two sub-bands, the filters are 2n+1 odd-order filters and the polyphase components are all of the same n degree. The bank is then PR if the determinant of the polyphase matrix is a monome. In other words if:

$$\mathrm{Det}\ \mathrm{H}_{[H_0,H_1]}(z) = H_{0,0}(z)H_{1,1}(z) - H_{1,0}(z)H_{0,1}(z) = \beta z^{-n}\ ; \qquad (11)$$

with β as a non-null constant.

Let $\hat{Q}$ *(z)* be the mirror filter of the mth order filter *Q(s)*, namely: $\hat{Q}(s) = z^{-m}Q(z^{-1})$. The orthogonal bank is characterized by filters $H_0(z)$ and $H_1(z)$ said to be quadrature conjugates, which can be written as follows $H_1(z) = \pm z^{-(2n+1)}H_0(z^{-1})$. Hereinafter, unlike the presentation given in [4], we choose the writing with a plus sign. Thus, the polyphase components meet the following relationship:

$$H_{0,1}(z) = -\hat{H}_{1,0}(z)\ ,\ \ H_{1,1}(z) = \hat{H}_{0,0}(z)\ . \qquad (12)$$

Consequently, the polyphase matrix can be written as follows:

$$\mathrm{H}_{[H_0,H_1]}(z) = \begin{bmatrix} H_{0,0}(z) & -\hat{H}_{1,0}(z) \\ H_{1,0}(z) & \hat{H}_{0,0}(z) \end{bmatrix}, \qquad (13)$$

and the relationship (11) can be written in the form:

$$\mathrm{Det}\ \mathbf{H}_{[H_0,H_1]}(z) = H_{0,0}(z)\hat{H}_{0,0}(z) + H_{1,0}(z)\hat{H}_{1,0}(z) = \beta z^{-n}\ , \quad (14)$$

The output signal is then such that $\hat{X}\ (s) = \gamma z^{-(2n+1)}X(z)$ with γ as a non-null constant. The matrix $H_{[H_0,H_1]}$ (z) is para-unitary and can be cascaded with [4]:

$$\mathbf{H}_{[H_0.H_1]}(z) = g\mathbf{A}(\alpha_n)\Lambda(z)\mathbf{A}(\alpha_{n-1})\ldots\Lambda(z)\mathbf{A}(\alpha_0). \quad (15)$$

where g is a non-null standardization constant, α_0, ..., α_n are n+1 real numbers and where the matrices A(α) for α real and Λ(z) are defined by:

$$\mathbf{A}(\alpha) = \begin{bmatrix} 1 & \alpha \\ -\alpha & 1 \end{bmatrix}\ ,\ \Lambda(z) = \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \quad (16)$$

An embodiment of a synthesis bank based on this mathematical writing produces the lattice depiction of Figure 4.

2. General characteristics of the case of the linear phase matched pair for M=4

For the linear phase matched pair, we necessarily have $F_T(z)$ and $F_R(z)$ symmetrical such that $F_T(z) = F_R(z)$, which can be written with only one expression $F(z)$. Let N be the order of $F(z)$. We therefore have:

$$F(z) = \sum_{k=0}^{N} f_k z^{-k} \quad (17)$$

The produced filter $P(z)$ is then expressed by:

$$P(z) = F^2(z) = \sum_{k=0}^{2N} p(k) z^{-k} \quad (18)$$

with of course $p(k) = p(2N - k)$. $P(z)$ meets the zero ISI condition (5) if:

$$p(k) = 0 \text{ for } k - N = 4l,\ l \neq 0 \quad (19)$$

It is possible to make a first observation with regard to the $N = 4n$ order filters.

Proposition 1 - *Let $F(z)$ be a $4n$ order symmetrical filter. $F(z)$ cannot be a null ISI filter.*

In order that $F(z)$, a symmetrical filter, may be a null ISI filter, we must have $f_0 \neq 0$. The highest degree monome of $P(z)$ is f_0^2 z^{-8n} while its 4n degree central term is z^{-1}. Since the difference in degrees is a multiple of 4, $F(z)$ is not at null ISI.

The polyphase decomposition of $F(z)$ for $M = 4$ can be written in the following form:

$$F(z) = F_0(z^4) + z^{-1}F_1(z^4) + z^{-2}F_2(z^4) + z^{-3}F_3(z^4)\ . \qquad (20)$$

with

$$F_l(z) = \sum_{n=0}^{n_l} f_{4n+l}z^{-n} \qquad (21)$$

To carry out the analysis of the different possible cases as a function of the Nth order, the property of symmetry of $F(z)$ and the fact that $4n_l + l \leq N$ will be used.

- If $N = 4n$, the degree of $F_0(z)$ is equal to n and that of the values $F_i(z),\ i = 1, 2, 3$ is strictly below n. According to Proposition 1, it is known that this case is of no use whatsoever for our problem.

- If $N = 4n + 1$, $F_1(z)$ is an n degree value, $F_0(z)$ is a $\leq n$ degree value while $F_2(z)$ and $F_3(z)$ are $\leq n - 1$ degree values. $F_1(z)$ then corresponds by mirror symmetry to $F_0(z)$. $F_1(z)$ is therefore actually an n degree value. We therefore have:

$$F_0(z) = \hat{F}_1(z) \text{ et } F_3(z) = \hat{F}_2(z) \qquad (22)$$

- If $N = 4n + 2$, we get $n_2 = n,\ n_1 = n$ and $n_3 = n - 1$. $F_2(z)$ is symmetrical to $F_0(z)$. We therefore have:

$$F_2(z) = \hat{F}_0(z)\ ,\quad F_1(z) = \hat{F}_1(z)\ ,\quad F_3(z) = \hat{F}_3(z) \qquad (23)$$

- If $N = 4n + 3$, $4n_l + 1 \leq 4n + 3$, $\forall l\ (0 \leq l \leq 3)$, and the filters F_i are all $\leq n$ degree

filters. Indeed, the highest-degree term of $F_0(z)$ is the $4n + 3$ degree term giving an n degree term at $F_3(z)$. The highest-degree term of $F_3(z)$ corresponds by symmetry to the constant term of $F_0(z)$. We therefore have:

$$F_3(z) = \hat{F}_0(z) \text{ et } F_2(z) = \hat{F}_1(z) \qquad (24)$$

Note: In the equation (22), we have deliberately written $F_0(z) = \hat{F}_1(z)$ and not $F_1(z) = \hat{F}_0(z)$ because it is certain that the degree of $F_1(z)$ is n since the constant term of $F_0(z)$ is not zero but not that the degree of $F_0(z)$ is n. Furthermore, this case where $f_1 = f_{N-1} = 0$ is considered further below.

The product $F^2(z)$ is then developed in taking account of the decomposition (20) and the following characterization of the filters $F(z)$ verifying the null ISI property is obtained.

Theorem 1 - *Let F(z) be an Nth order symmetrical filter verifying* $F(z = 0) \neq 0$. *Then, depending on the values of N, F(z) is at null ISI if and only if:*

- $N = 4n + 1$,

$$F_1(z)\hat{F}_1(z) + z^{-1}F_2(z)\hat{F}_2(z) = \gamma z^{-n}\ , \qquad (25)$$

- If $N = 4n + 2$,

$$2F_0(z)\hat{F}_0(z) + F_1^2(z) + z^{-1}F_3^2(z) = \gamma z^{-n}\ , \qquad (26)$$

- If $N = 4n + 3$,

$$F_0(z)\hat{F}_0(z) + F_1(z)\hat{F}_1(z) = \gamma z^{-n}\ , \qquad (27)$$

where γ is a non-null constant.

The following theorem shows that all the $N = 4(n - 1) + 3$ order solutions are obtained from $N = 4n + 1$ order solutions.

Theorem 2 - *Let F(z) be a null ISI symmetrical filter of the $4n + 1$, $n \geq 1$ order. Then if $F_i(z)$, $i = 0, ..., 3$ designates the polyphase components, $F_0(z)$ is of a degree strictly*

below n and $F_1(z)$ can be written in the following form: $F_1(z) = z^{-1}K_1(z)$. We then have $\hat{K}_1(z) = F_0(z)$ and the filter $\overline{F}(z)$ with polyphase components $[F_0(z), F_2(z), F_3(z), K_1(z)]$ is a symmetrical $4(n-1)+3$ order, null ISI filter.

Demonstration - This immediately results from the relationship (25) since the constant term of the member on the left is null. We therefore have

$F_1(z=0)\,\hat{F}_1(z=0) = f_0 f_1 = 0$. Since $F_1(z)$ contributes with f_0 to the highest-degree term, $f_0 \neq 0$ and therefore $f_1 = 0$. Writing $F_1(z)$ in the form $F_1(z) = z^{-1}K_1(z)$ and knowing that $\hat{F}_1(z) = \hat{K}_1(z)$, the relationship (26) can also be written as follows:

$$z^{-1}K_1(z)\hat{K}_1(z) + z^{-1}F_2(z)\hat{F}_2(z) = \gamma z^{-n}, \tag{28}$$

Since $\hat{K}_1(z) = F_0(z)$ and therefore $K_1(z) = \hat{F}_0(z)$, we obtain the relationship (27) for a filter where $K_1(z)$ is substituted for $F_3(z)$ and therefore has the polyphase components: $[F_0(z), F_2(z), F_3(z), K_1(z)]$. It is therefore possible, in one example, to verify the passage from a 9th order filter to a 7th order filter.

Example

$$\begin{aligned} F_0(z) &= f_0 + f_4 z^{-1} + f_1 z^{-2} &&= f_0 + f_4 z^{-1} \\ F_1(z) &= f_1 + f_4 z^{-1} + f_0 z^{-2} &&= z^{-1}(f_4 + f_0 z^{-1}) \quad = z^{-1}K_1(z) \\ F_2(z) &= f_2 + f_3 z^{-1} \\ F_3(z) &= f_3 + f_2 z^{-1} \end{aligned} \tag{29}$$

The composite filter of $[F_0(z), F_2(z), F_3(z), K_1(z)]$ is a 7th order filter and is then expressed by:

$$f_0 + f_2 z^{-1} + f_3 z^{-2} + f_4 z^{-3} + f_4 z^{-4} + f_3 z^{-5} + f_2 z^{-6} + f_0 z^{-7} \tag{30}$$

A third theorem shows that the $N = 4n + 3$ order solutions are obtained from the orthogonal two sub-band filter bands.

Theorem 3 - *Let $[H_0(z), H_1(z)]$ be a bank of $2n + 1$ order orthogonal filters with two sub-bands whose polyphase matrix is given by (13). Then the filter $\bar{H}(z)$ for which the four polyphase components are $[H_{0,0}(z), H_{1,0}(z), \hat{H}_{1,0}(z), \hat{H}_{0,0}(z)]$ is a linear phase, $4n + 3$ order, null ISI filter. Reciprocally, all the filters of this type are obtained from orthogonal filter banks with two sub-bands.*

Demonstration - The demonstration is immediate since the $4n + 3$ order linear phase, null ISI filters are characterized by the relationship (27) which, in the terms of the theorem, becomes equivalent to (14).

APPENDIX 3

$N = 4n + 2$ order Nyquist pair

1 - Initial factorization

Theorem 1 in Appendix 2 gives us, inter alia, the condition that a $4n + 2$ order $F(z)$ filter must meet in order to be at null ISI. It may be recalled that it is written as follows:

$$2F_0(z)\hat{F}_0(z) + F_1^2(z) + z^{-1}F_3^2(z) = \gamma z^{-n} , \qquad (31)$$

where γ is a non-null constant and where the components $F_i(z)$ are all nth degree components apart from $F_3(z)$, and meet the relationships $\hat{F}_2(z) = F_0(z)$, $F_1(z) = \hat{F}_1(z)$ and $F_3(z) = \hat{F}_3(z)$.

Theorem 4 - *Let a_i, $i = 1, \ldots, n + 1$, $n \geq 0$, $n + 1$ be constant values. We consider the column vector $[F_0(z), F_1(z), F_2(z), F_3(z)]^T$ defined by the following equality:*

$$\begin{bmatrix} F_0(z) \\ F_1(z) \\ F_2(z) \\ F_3(z) \end{bmatrix} = M(a_1)\ Z\ M(a_2)\ Z\ \ldots\ Z\ M(a_n)\ Z \begin{bmatrix} 1 \\ a_{n+1} \\ 1 \\ 0 \end{bmatrix} , \qquad (32)$$

Then the filter F(z) whose polyphase components are the values $F_i(z)$, $i = 1, \ldots, 4$, is a null ISI, 4n +2 order, monic, linear phase filter.

Reciprocally, any null ISI, linear phase, monic, 4n + 2 order filter accepts a decomposition of this form.

Demonstration - For $n = 0$, the polyphase components are the constant polynomials $F_0(z) = 1$, $F_1(z) = a_1$, $F_2(z) = 1$ and $F_3(z) = 0$. These polynomials trivially verify the condition (31) as well as the conditions of symmetry and degree. We then have $F(z) = 1 + a_1z^{-1} + z^{-2}$. Let us now suppose that the components of a $4n + 2$ order $F(z)$ filter $F_i(z)$, $i = 0, \ldots, 3$ verify the condition (31) as well as the conditions of symmetry and degree. Let us build $L_i(z)$, $i = 0, \ldots, 3$ with:

$$\begin{bmatrix} L_0(z) \\ L_1(z) \\ L_2(z) \\ L_3(z) \end{bmatrix} = \mathrm{M}(a_1)\ \mathrm{Z} \begin{bmatrix} F_0(z) \\ F_1(z) \\ F_2(z) \\ F_3(z) \end{bmatrix} . \qquad (33)$$

We have:

$$L_0(z) = F_0(z) - \frac{a_1^2}{2}z^{-1}F_2(z) + a_1z^{-1}F_3(z) , \qquad (34)$$

$$L_1(z) = -a_1 F_0(z) - a_1 z^{-1} F_2(z) + (1 - \frac{a_1^2}{2}) z^{-1} F_3(z) \; , \quad (35)$$

$$L_2(z) = -\frac{a_1^2}{2} F_0(z) + z^{-1} F_2(z) + a_1 z^{-1} F_3(z) \; , \quad (36)$$

$$L_3(z) = (1 + \frac{a_1^2}{2}) F_1(z) \; . \quad (37)$$

The verification of the conditions of degree of symmetry on the components $L_i(z)$, i = 0, ..., 3 is done almost immediately. By developing the analogous expression of (31) for the components $L_i(z)$, i = 0, ..., 3, we obtain:

$$2L_0(z)\hat{L}_0(z) + L_1^2(z) + z^{-1}L_3^2(z) = \frac{(2+a_1^2)^2}{4} z^{-1} \left(2F_0(z)\hat{F}_0(z) + F_1^2(z) + z^{-1}F_3^2(z)\right) \quad (38)$$

According to (31), the direct part of the theorem is therefore demonstrated.

The reciprocal is demonstrated if it can be established that the relationships (34-37) can uniquely determine a coefficient a_1 and the components $F_i(z)$, i = 0, ..., 3 of a null ISI, linear phase, monic $F(z)$ filter. Since we should have $F_0(z = 0) = 1$ (since $F(z)$ is monic and symmetrical, its constant term is equal to 1), according to (35) we have $a_1 = -L_1(z = 0)$. If $a_1 = 0$, then we are in the particular case where $L_0(z)$ is an n degree term, the $n + 1$ degree term being zero, the symmetrical polynomial $L_1(z)$ has a null constant term and its highest degree term is null, and finally the constant term of $L_2(z)$ is null (but not its $n + 1$ degree term whose coefficient is 1). The relationships (34-37) are then inverted by $F_0(z) = L_0(z)$, $F_1(z) = L_3(z)$, $F_2(z) = L_2(z)$ quo z^{-1} and $F_3(z) = L_1(z)$ quo z^{-1}. It is now assumed that $a_1 \neq 0$ but this is not actually necessary. The formal inversion of the formulae (34-37) leads to:

$$F_0(z) = \frac{4}{(2+a_1^2)^2}\left(L_0(z) - a_1 L_1(z) - \frac{a_1^2}{2} L_2(z)\right) \; , \quad (39)$$

$$F_1(z) = \frac{2}{2+a_1^2} L_3(z) \; ; \quad (40)$$

$$F_2(z) = \frac{4}{(2+a_1^2)^2}\left(-\frac{a_1^2}{2} L_0(z) - a_1 L_1(z) + L_2(z)\right) \text{ quo } z^{-1} \; , \quad (41)$$

$$F_3(z) = \frac{4}{(2+a_1^2)^2}\left(a_1 L_0(z) + (1 - \frac{a_1^2}{2}) L_1(z) + a_1 L_2(z)\right) \text{ quo } z^{-1} \quad (42)$$

It is then enough to show that the quotients by z^{-1} are exact. Since $F(z)$ is monic, $L_0(z = 0)$ =

1, $L_1(z = 0) = -a_1$ by defining of a_1 and according to the relationship (31), $L_2(z = 0) = a_1^2/2$. The straight line members of the equalities defining $F_2(z)$ and $F_3(z)$ therefore cancel out in $z = 0$ and the quotients are exact. It can then be ascertained that the polynomials F_i, $i = 0, \ldots, 3$ meet the appropriate conditions of symmetry.

2 - Equivalent decomposition

Noting the following matrix as $\mathbf{R}(\alpha)$:

$$\mathrm{R}(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{43}$$

and using the formula (32) we get:

$$\begin{bmatrix} F_0(z) \\ F_1(z) \\ F_2(z) \\ F_3(z) \end{bmatrix} = \mathrm{P}^T\,\mathrm{R}(a_1)\,\mathrm{PZP}^T\,\mathrm{R}(a_2)\,\mathrm{PZP}^T \;\ldots\; \mathrm{PZP}^T\,\mathrm{R}(a_n)\,\mathrm{PZ} \begin{bmatrix} 1 \\ a_{n+1} \\ 1 \\ 0 \end{bmatrix}$$

In the sending diagram, Ma is used to denote the block associated with the right side vector.

The matrix $\bar{\mathrm{Z}}$ is then introduced. This matrix is defined by:

$$\bar{\mathrm{Z}} = \mathrm{PZP}^T = \begin{bmatrix} \frac{1}{2}(1+z^{-1}) & 0 & \frac{1}{2}(-1+z^{-1}) & 0 \\ 0 & 0 & 0 & z^{-1} \\ \frac{1}{2}(-1+z^{-1}) & 0 & \frac{1}{2}(1+z^{-1}) & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \tag{44}$$

taking account of:

$$\mathrm{P}\begin{bmatrix} 1 \\ a_{n+1} \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \sqrt{2} \\ a_{n+1} \\ 0 \\ 0 \end{bmatrix}, \tag{45}$$

we obtain, with a multiplier factor g, the breakdown:

$$\begin{bmatrix} F_0(z) \\ F_1(z) \\ F_2(z) \\ F_3(z) \end{bmatrix} = g\mathrm{P}^T\mathrm{R}(a_1)\,\bar{\mathrm{Z}}\,\mathrm{R}(a_2)\,\bar{\mathrm{Z}}\,\ldots\,\mathrm{R}(a_n)\,\bar{\mathrm{Z}} \begin{bmatrix} 1 \\ a_{n+1} \\ 0 \\ 0 \end{bmatrix}. \tag{46}$$

Operational complexity

The equation (46) directly gives the structure of the making of the half-Nyquist sending and reception filters. It corresponds to a system with four inputs and four outputs which, in the diagrams of Figures 6 and 7, take the place of the set of four polyphase filters. It will be noted that, in the case of the sending filter, the outputs have to be inverted with respect to the writing (46) which on the contrary corresponds precisely to that of the reception filter. Apart

from this difference of detail, the structure for each of these $4n + 2$ order filters will therefore correspond to:

- n matrix blocks of the $\bar{\bar{Z}}$ and $\mathbf{R}(a_l)$ types,
- one $\mathbf{P}^T$ matrix block,
- one vector block $[1, a_{n+1}, 0, 0]^T$,
- a multiplier g which is to be taken into account only for sending or reception.

Let us now make a more detailed examination of the cost of making each of these elements.

- Matrices $\mathbf{R}(\alpha_l)$

The matrices $\mathbf{R}(\alpha_l)$, $1 \leq l \leq n$ are block matrices which can take the form:

$$R(\alpha_l) = \begin{pmatrix} G_l & 0 \\ 0 & I \end{pmatrix} \tag{47}$$

where $\mathbf{I}$ is the unity matrix and $\mathbf{G}_l$ is the matrix of rotation for the angle α_l, namely:

$$G_l = \begin{pmatrix} \cos(\alpha_l) & -\sin(\alpha_l) \\ \sin(\alpha_l) & \cos(\alpha_l) \end{pmatrix} \tag{48}$$

The circuit arrangement cost is therefore quite simply that of the lattice corresponding to the rotation $\mathbf{G}_l$, namely four multiplications and two additions.

- The matrix $\bar{Z}$.

It can be used for the association, with an input vector e, of the output vector s by the relationship s = $\bar{Z}$e which is expressed by the following four equations:

$$\begin{aligned} s_0 &= \tfrac{1}{2}[(e_0 - e_2) + z^{-1}(e_0 + e_2)] \\ s_1 &= z^{-1}e_1 \\ s_2 &= \tfrac{1}{2}[(e_2 - e_0) + z^{-1}(e_0 + e_2)] \\ s_3 &= e_3 \end{aligned} \tag{49}$$

The structural diagram of a block of this kind is shown in Figure 14. It can be specified that the multiplications by 1/2 can be reduced through a simple shift of binary data. In addition to these two shifts, the cost is therefore simply four additions and one sign inversion.

- The matrix $\mathbf{P}^T$

It takes the form:

$$\mathbf{P}^T = \begin{pmatrix} a & 0 & -a & 0 \\ 0 & 1 & 0 & 0 \\ a & 0 & a & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{50}$$

with $a = \dfrac{1}{\sqrt{2}}$. This embodiment can also be deduced from the input/output equations which in the present case are written as follows:

$$\begin{aligned} s_0 &= a(e_0 + e_2) \\ s_1 &= e_1 \\ s_2 &= a(-e_0 + e_2) \\ s_3 &= e_3 \end{aligned} \tag{51}$$

immediately it is therefore possible to then deduce the diagram of Figure 15.

The operational cost is two multiplications, two additions and one sign inversion.

- The vector block $[1, a_{n+1}, 0, 0]^T$

Its circuit arrangement cost is the equivalent of only one single multiplication.

The full circuit arrangement diagram according to the equation (46) therefore corresponds to a system of four inputs and four outputs as shown in Figure 16. Naturally, this system is found at sending and at reception. The order of complexity of each of these filters is therefore $4n$ MPU and $6n$ APU.

Thus, as already stated, at sending it is necessary to invert the outputs. In the drawing showing the making of the device in Figure 17, this operation is symbolized by the block **J** corresponding to the antidiagonal matrix, namely:

$$\mathbf{J} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \tag{52}$$

The reception diagram for the $4n + 2$ order is shown in Figures 18 and 19. It is deduced from the following computations.

The matrix $\hat{\mathrm{Z}}$ is introduced, defined by :

$$\hat{\mathrm{Z}} = \begin{bmatrix} \frac{1}{2}(1+z^{-1}) & 0 & \frac{1}{2}(1-z^{-1}) & 0 \\ 0 & 0 & 0 & z^{-1} \\ \frac{1}{2}(1-z^{-1}) & 0 & \frac{1}{2}(1+z^{-1}) & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

It is the matrix obtained in taking the mirror elements of the elements of the matrix $\bar{\mathrm{Z}}$. We have:

$$\bar{\mathrm{Z}}\,\hat{\mathrm{Z}} = z^{-1}\mathrm{I}_4\,,$$

where $\mathbf{I}_4$ is the 4th order identity matrix.

Then the following identities are verified:

$$\mathrm{P}\,\mathrm{P}^T = \mathrm{I}_4 \ , \ \ \mathrm{R}(a)^{-1} = \mathrm{R}(a)^T = \mathrm{R}(-a)$$

for any value of a.

By noting the matrix product as **M**:

$$\mathrm{M} = \mathrm{P}^T\mathrm{R}(a_1)\bar{\mathrm{Z}}\ldots\mathrm{R}(a_n)\bar{\mathrm{Z}}$$

and the matrix product as **N :**

$$\mathrm{N} = \hat{\mathrm{Z}}\mathrm{R}(-a_n)\ldots\hat{\mathrm{Z}}\mathrm{R}(-a_1)\mathrm{P}$$

we therefore have:

$$\mathrm{NM} = z^{-n}\mathrm{I}_4 \ ,$$

and consequently :

$$\mathrm{N}\begin{bmatrix} F_0(z) \\ F_1(z) \\ F_2(z) \\ F_3(z) \end{bmatrix} = gz^{-n}\begin{bmatrix} 1 \\ a_{n+1} \\ 0 \\ 0 \end{bmatrix} .$$

We then introduce the matrix **K** defined by:

$$\mathrm{K} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} .$$

Figure 19 gives the drawing for the making of the block corresponding to the matrix **N** and Figure 20 is the for the making of the reception filter resulting therefrom. If the constant g is equal to $1/(1 + a_{n+1})$, then a signal transmitted by the sending system of Figure 17 and then received by the system of Figure 20 produces an identical signal with a delay of n + 2 samples.

APPENDIX 4
Single-lattice reception diagram

If we assume a back-to-back operation, it is indeed possible to simplify the diagram of Figure 10 by using the equation that fulfils a null ISI. The single-lattice diagram of Figure 10a is then obtained. In this case, only the null ISI and the phase linearity of the sending filter are provided.

The validity of this drawing of Figure 10a can be verified from the equation (37) of the patent after multiplication by z^{-1}. For a so-called back-to-back system, the input signal of the reception filter is given by:

$$Y(z) = z^{-1}[F_0(z^4) + z^{-1}F_1(z^4) + z^{-2}\hat{F}_1(z^4) + z^{-3}\hat{F}_0(z^4)]X(z^4) \quad (53)$$

Referring to Figure 10a, the signals after decimation are given, from top to bottom, by:

$$\begin{array}{ll} Y(z)\downarrow_4 = z^{-1}\hat{F}_0(z)X(z), & z^{-1}Y(z)\downarrow_4 = z^{-1}\hat{F}_1(z)X(z), \\ z^{-2}Y(z)\downarrow_4 = z^{-1}F_1(z)X(z), & z^{-3}Y(z)\downarrow_4 = z^{-1}F_0(z)X(z) \end{array} . \quad (54)$$

At input of the direct lattice, the signals are therefore given, from top to bottom, by:

$$z^{-1}[\hat{F}_0(z) + \hat{F}_1(z)]X(z) \text{ et } z^{-1}[F_0(z) - F_1(z)]X(z).$$

The output signal is therefore:

$$S(z) = 2gz^{-1}[F_0(z)\hat{F}_0(z) + F_1(z)\hat{F}_1(z)]X(z) = 2gz^{-1}[\gamma z^{-n}], \quad (55)$$

with

$$\gamma = \prod_{i=0}^{n}(1+\alpha_i^2).$$

It is thus verified that for $g = 1/\gamma$, the output signal is identical to the input signal except for the $n + 1$ sample delay.

APPENDIX 5

The case of the $N = 4n + 1$ order

Preliminary observation - To preserve a lattice with n + 1 cells with coefficients a_i, with $0 \leq i \leq n$, as defined for the 4n + 3 order, we present the case of the $4n + 5$ order.

Let L_i, $i = 0, ..., 3$ be the polyphase components of a null ISI $4n + 3$ order $L(z)$ filter. It is possible to associate, with this filter, a lattice built as in Figure 8a of the patent with a set of coefficients α_i, $i = 0, ..., n$. We have $L_2(z) = \hat{L}_1(z)$ and $L_3(z) = \hat{L}_0(z)$.

The filter $F(z)$ whose components F_i i=0,...,3 are defined by

(56) $$F_0(z) = L_0(z) ,$$

(57) $$F_1(z) = z^{-1}\hat{L}_0(z) ,$$

(58) $$F_2(z) = L_1(z) ,$$

(59) $$F_3(z) = \hat{L}_1(z) ,$$

is a null ISI, $4n + 5$ order linear phase filter. Furthermore, the Theorem 2 in Appendix 2 of the patent establishes the fact that such filters are necessarily obtained in this way. Of course, the synthesis of such a filter is done so as to optimize the coefficients α_i, $i = 0, ..., n$ to obtain the right frequency specifications for the filter $F(z)$, not for the filter $L(z)$! This therefore leads to the obtaining of diagrams for sending and reception associated with $F(z)$ from the lattice built with the coefficients α_i and the associated inverted lattice.

The type I polyphase breakdown applied to this filter is expressed by the following expression:

$$F(z) = L_0(z^4) + z^{-1}[z^{-4}\hat{L}_0(z^4)] + z^{-2}L_1(z^4) + z^{-3}\hat{L}_1(z^4) \quad (60)$$

It can be ascertained that, for the sending part, this equation may be expressed by the lattice diagram of Figure 20 wherein a delay is applied. This delay is given by z^{-3} at output of $F(z)$.

For the reception part as in the case of the $4n + 3$ order, it is possible to make a 4-lattice system that ensures the totality of the properties sought.

Similarly, simplifications appear if we consider the system known as the back-to-back system. The input signal of the reception filter can then be written as follows:

$$Y(z) = [z^{-3}L_0(z^4) + z^{-8}[z^{-4}\hat{L}_0(z^4)] + z^{-5}L_1(z^4) + z^{-6}\hat{L}_1(z^4)]X(z^4) \quad (61)$$

To recover the input signal, apart from a delay, we then implement the device of Figure 21.

Referring to Figure 21, the signals after this decimation are, from top to bottom, given by:

$$\begin{array}{cc} Y(z)\downarrow_4 = z^{-2}\hat{L}_0(z)X(z), & z^{-1}Y(z)\downarrow_4 = z^{-1}L_0(z)X(z), \\ z^{-2}Y(z)\downarrow_4 = z^{-2}\hat{L}_1(z)X(z), & z^{-3}Y(z)\downarrow_4 = z^{-2}L_1(z)X(z) \end{array} \cdot \quad (62)$$

From top to bottom, the signals at input of the lattice are respectively given by:

$$z^{-2}[\hat{L}_0(z) + \hat{L}_1(z)]X(z) \text{ et } z^{-2}[L_0(z) - L_1(z)]X(z).$$

Thus, as in the case of the $4n + 3$ order but this time with a processing delay of $n + 2$ samples, we have an output signal which, for $g = 1/(2\gamma)$ is identical to that of the input.

APPENDIX 6

1 - The evaluation of the complexity of the reception filters

Two cases have to be considered depending on whether it is sought or not to keep all the properties sought. Thus, for the $4n + 3$ and $4n + 5$ order filters:

- To keep null ISI, the matched pair, the phase linearity of the sending and reception filters, as shown in Figures 10, four lattices are used. These four lattices work at the lowest rate. Given the fact that for each lattice two multiplications and additions are necessary, except for each first cell (one multiplier and one adder), the operational complexity is (8n + 4) MPU and APU, plus a sign inversion.

- For a back-to-back system, simplifications are possible but only null ISI and the phase linearity of the sender filter are guaranteed. For the simplified $4n + 3$ and 4n + 5 order reception filters, the operational complexity corresponds then to that of a single lattice plus that of the three adders, the multiplier by g and the inverter. The computations performed at the lowest rate can therefore be estimated at $(2n + 3)$ MPU, $(2n + 5)$ APU and one inversion.

2 - A method of generic synthesis

As compared with the initially proposed method of synthesis, we now have a synthesis method that can be used, for all three types of solutions, to meet a frequency specification such as the one shown in Figure 2.

For the filter $F(z)$ with a length L, different from $4n + 1$, in other words for an N order different from $4n$ and for a fixed value of the fall-back factor ρ, we seek to minimize the cost function:

$$\Phi(F) = \sup\{w_P\,(1 - |F(1)|)^2\,, w_C\left(\frac{\sqrt{2}}{2} - |F(e^{j\frac{\pi}{8}})|\right)^2, w_S \sup_{[\omega_S,\pi]} |F(e^{j\omega})|^2\}.$$

We use the FSQP (feasible sequential quadratic programming) algorithm developed by the team led by A.L. Tits [5] to search for the minimum of a set of non-linear even cost functions subjected to general, even and non-linear constraints. Since this method is a method of local optimization, the choice of an initial point is necessary. For a given weight w in the stopping band and a weight equal to 1 at the starting point and in the Nyquist frequency, namely $\pi/8$, we calculate an initial filter F^{init} that is optimal for the norm of the minimal value. This filter F^{init} is then used for direct computation of a set of lattice coefficients for a filter

producing a matched pair with null ISI. Since F^{init} does not itself produce a matched pair with null ISI, the identification between the two sets of coefficients, the transversal coefficients of F^{init} and the lattice coefficients is not exactly achieved. We therefore obtain a different filter, referenced F^{init}, whose lattice coefficients are considered to be the initial point of the optimization problem described by (63). For this problem, we fix $w_P = 1$, $w_C = 2$, $w_S = 0.5$.

Furthermore, for any lattice structure optimized for a given value ρ_0, we also use a continuation algorithm to obtain a lattice structure optimized for a value ρ_1, that is different in taking a sequence of intermediate values for ρ: the result of the lattice structure optimized for a value of ρ in the sequence is the initial point for the following value of ρ.

Figure 22 shows a higher attenuation in the stopping band than could be obtained for a fixed length and fall-back factor.

Let us consider for example the values $L = 43$ and $\rho = 0.5$: the best attenuation obtained by direct optimization is equal to 52.25 dB. The frequency response of the corresponding sending filter F^{opt} is given by Figure 23. The transversal coefficients and the coefficients of the lattice structure are given in Table 3.

Transversal coefficients

f_0	$-1.033698\ 10^{-4}$	f_{11}	$8.074275371845\ 10^{-3}$
f_1	$-3.6954272174\ 10^{-4}$	f_{12}	$1.5774357742194\ 10^{-2}$
f_2	$6.60549905248\ 10^{-4}$	f_{13}	$1.2603068512975\ 10^{-2}$
f_3	$-1.09979829792\ 10^{-4}$	f_{14}	$-6.358804372375\ 10^{-3}$
f_4	$4.31563793685\ 10^{-4}$	f_{15}	$-3.3366501346017\ 10^{-2}$
f_5	$6.08998879046\ 10^{-4}$	f_{16}	$-4.6538674170018\ 10^{-2}$
f_6	$6.39123817678\ 10^{-4}$	f_{17}	$-2.1044564354449\ 10^{-2}$
f_7	$-7.82626716253\ 10^{-4}$	f_{18}	$5.1214636348857\ 10^{-2}$
f_8	$-3.761395765875\ 10^{-3}$	f_{19}	$1.51186813816588\ 10^{-1}$
f_9	$-4.983816032565\ 10^{-3}$	f_{20}	$2.39358058463701\ 10^{-1}$
f_{10}	$-9.24070721312\ 10^{-4}$	f_{21}	$2.74720061562686\ 10^{-1}$

Lattice coefficients

α_1	-3.1848708937
α_2	$2.648518499\ 10^{-1}$
α_3	-2.9385797132
α_4	$-9.349457966\ 10^{-1}$
α_5	$9.246893837\ 10^{-1}$
α_6	$-1.308356726\ 10^{-1}$
α_7	3.6683584963
α_8	$-6.126257549\ 10^{-1}$
α_9	-1.2465410017
α_{10}	$-1.439677875\ 10^{-1}$
α_{11}	-3.5749582735

Table 3 : Lattice and transversal coefficients of the example with L = 43 and $\rho = 0.5$ ($f_{42-i} = f_i$)

APPENDIX 7

References

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CLAIMS

1. Method for the making of a digital Nyquist filter with null inter-symbol interference designed to process a physical signal transmitted between a sender and a receiver through a transmission channel,

said filter being an Nth order $P(z) = F^2(z)$ symmetrical filter implementing an oversampling factor M=4 and forming a matched pair comprising a sending filter (12) and a reception filter (15) whose polyphase breakdown of F(z) can be written as follows:

$$F(z) = F_0(z^4) + z^{-1}F_1(z^4) + z^{-2}F_2(z^4) + z^{-3}F_3(z^4)$$

characterized in that N is different from 4n, n being an integer:

and in that :

- If N=4n+1, $F_1(z)\hat{F}_1(z) + z^{-1}F_2(z)\hat{F}_2(z) = \gamma z^{-n}$
- If N=4n+2, $2F_0(z)\hat{F}_0(z) + F_1^2(z) + z^{-1}F_3^2(z) = \gamma z^{-n}$
- If N=4n+3, $F_0(z)\hat{F}_0(z) + F_1(z)\hat{F}_1(z) = \gamma z^{-n}$

$\hat{F}$ being the mirror symmetry of F and γ being a non-null constant.

2. Method according to claim 1, characterized in that N is equal to 4n+3 or 4n+1 and:

said sending filter (12) performs an interpolation (121) by a factor M = 4 and has a circuit arrangement corresponding to a polyphase breakdown known as the type II breakdown, such that:

$$_F F(z) = \begin{bmatrix} z^{-3} & z^{-2} & z^{-1} & 1 \end{bmatrix} \begin{bmatrix} \hat{F}_0(z^4) \\ \hat{F}_1(z^4) \\ F_1(z^4) \\ F_0(z^4) \end{bmatrix}$$

and said reception filter (15) performs a decimation (152) by a factor M = 4 and has a circuit arrangement corresponding to a polyphase breakdown known as the type I breakdown, such that:

$$F(z)=\left[F_0(z^4)\ F_1(z^4)\ \hat{F}_1(z^4)\ \hat{F}_0(z^4)\right]\begin{bmatrix}1\\ z^{-1}\\ z^{-2}\\ z^{-3}\end{bmatrix}$$

3. Method according to any of the claims 1 and 2, characterized in that, in said sending filter (12), a filtering step followed by a step of interpolation by a factor of M=4 is performed.

4. Method according to any of the claims 1 à 3, characterized in that, in said reception filter (15), a step of decimation by a factor M=4 is performed, followed by a filtering step.

5. Method according to any of the claims 1 to 4, characterized in that said sending filter (12) and/or said reception filter (15) have a structure in the form of at least one lattice.

6. Method according to claim 5, characterized in that said sending filter (12) and said reception filter (15) are each constituted by a pair of polyphase components respectively given by the following equations :

$$\begin{bmatrix}F_0\\ F_1\end{bmatrix}=gA(\alpha_n)\Lambda(z)A(\alpha_{n-1})\ldots\Lambda(z)A(\alpha_0)\begin{bmatrix}1\\ 0\end{bmatrix}$$

$$\begin{bmatrix}-\hat{F}_1\\ \hat{F}_0\end{bmatrix}=gA(\alpha_n)\Lambda(z)A(\alpha_{n-1})\ldots\Lambda(z)A(\alpha_0)\begin{bmatrix}0\\ 1\end{bmatrix}$$

with :

$$A(\alpha)=\begin{bmatrix}1 & \alpha\\ -\alpha & 1\end{bmatrix} \qquad \text{and} \qquad \Lambda(z)=\begin{bmatrix}1 & 0\\ 0 & z^{-1}\end{bmatrix}$$

where g is a non-null constant of standardisation and α_i are real coefficients.

7. Method according to claim 6, characterized in that it implements a two-lattice structure.

8. Method according to claim 6, characterized in that it implement a single-lattice structure working at a double frequency.

9. Device for the filtering of Nyquist digital signals with null inter-symbol interference designed to process a physical signal transmitted between a sender and a receiver through a transmission channel,

based on an Nth order $P(z) = F^2(z)$ symmetrical filter implementing an oversampling factor $M = 4$ and forming a matched pair comprising a sending filter (12) and a reception filter (15),

the polyphase breakdown of F(z) of this symmetrical filter being written as follows:

$$F(z) = F_0(z^4) + z^{-1}F_1(z^4) + z^{-2}F_2(z^4) + z^{-3}F_3(z^4)$$

characterized in that N is different from 4n, n being an integer,

- If N=4n+1, $F_1(z)\hat{F}_1(z) + z^{-1}F_2(z)\hat{F}_2(z) = \gamma z^{-n}$
- If N=4n+2, $2F_0(z)\hat{F}_0(z) + F_1^2(z) + z^{-1}F_3^2(z) = \gamma z^{-n}$
- If N=4n+3, $F_0(z)\hat{F}_0(z) + F_1(z)\hat{F}_1(z) = \gamma z^{-n}$

$\hat{F}$ being the mirror symmetry of F and γ being a non-null constant.

ABSTRACT

The invention pertains to a method for the making of a Nyquist digital filter with null inter-symbol interference, designed to process a physical signal transmitted between a sender and a receiver through a transmission channel,

said filter being a symmetrical N order $P(z) = F^2(z)$ filter implementing an oversampling factor M=4, and forming a matched fair comprising a sending filter and a reception filter,

the polyphase breakdown of F(z) being written as :

$$F(z) = F_0(z^4) + z^{-1}F_1(z^4) + z^{-2}F_2(z^4) + z^{-3}F_3(z^4)$$

characterized in that N is different from 4n, with n as an integer,

and in that :

- If N=4n+1, $F_1(z)\hat{F}_1(z) + z^{-1}F_2(z)\hat{F}_2(z) = \gamma z^{-n}$
- If N=4n+2, $2F_0(z)\hat{F}_0(z) + F_1^2(z) + z^{-1}F_3^2(z) = \gamma z^{-n}$
- If N=4n+3, $F_0(z)\hat{F}_0(z) + F_1(z)\hat{F}_1(z) = \gamma z^{-n}$

$\hat{F}$ being the mirror symmetry of F and γ being a non-null constant.

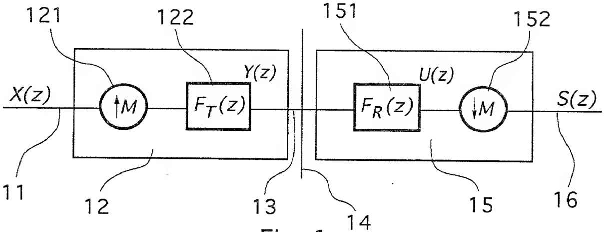


Fig. 1

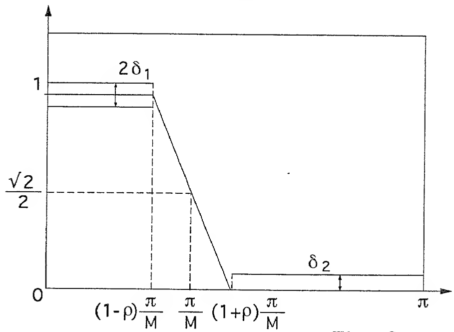


Fig. 2

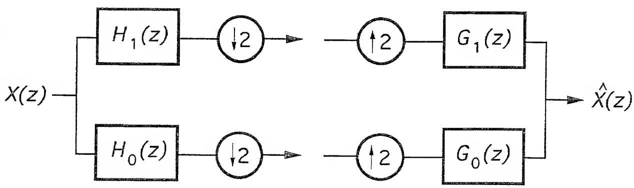


Fig. 3

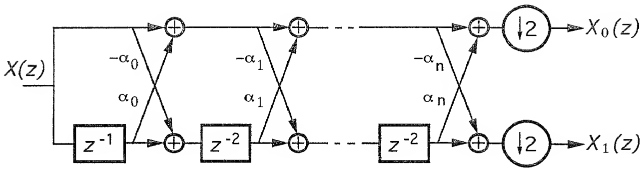


Fig. 4

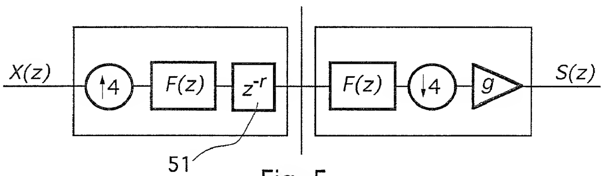


Fig. 5

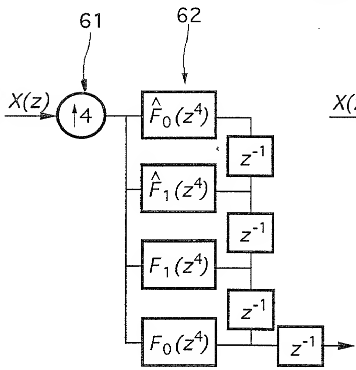


Fig. 6a

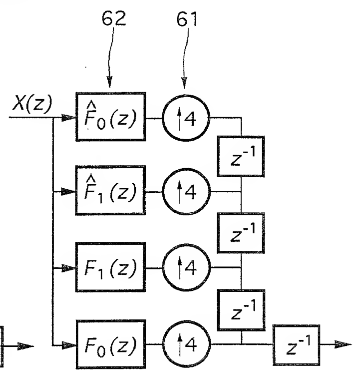


Fig. 6b

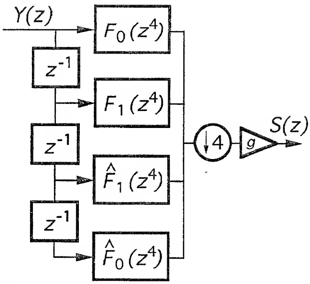


Fig. 7a

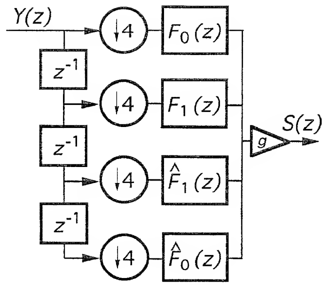


Fig. 7b

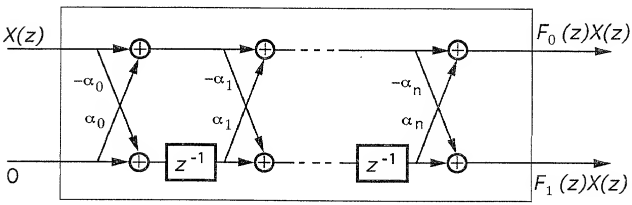


Fig. 8a

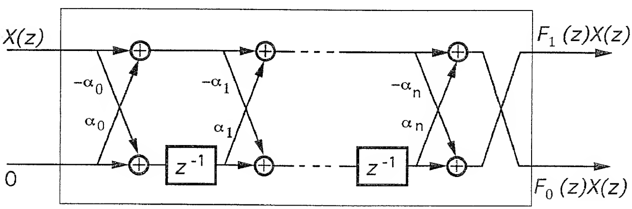


Fig. 8b

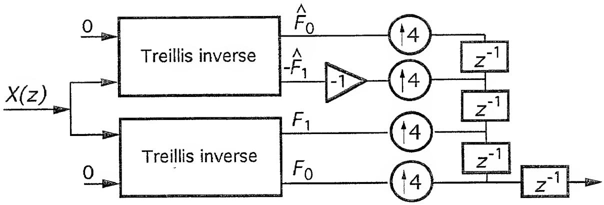


Fig. 9

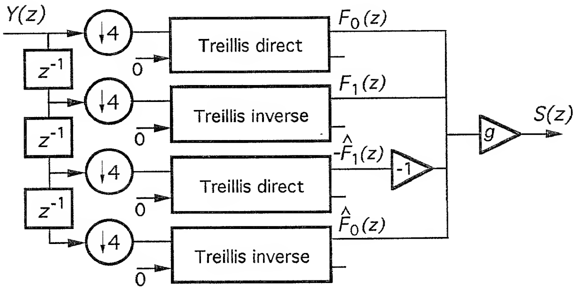


Fig. 10

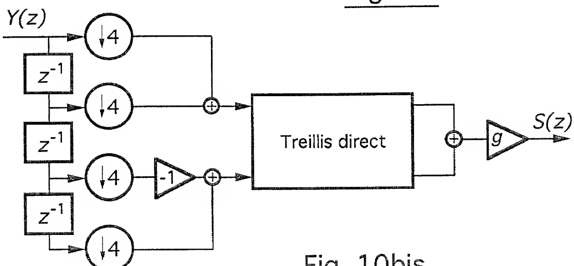


Fig. 10bis

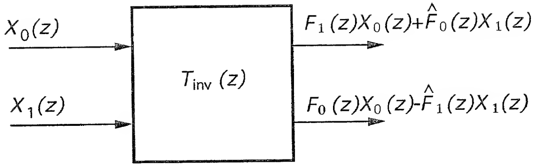


Fig. 11

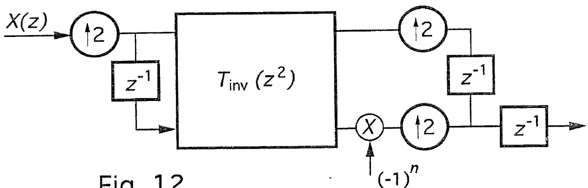


Fig. 12

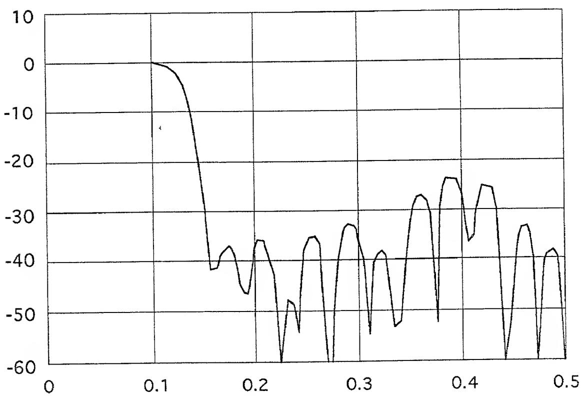


Fig. 13

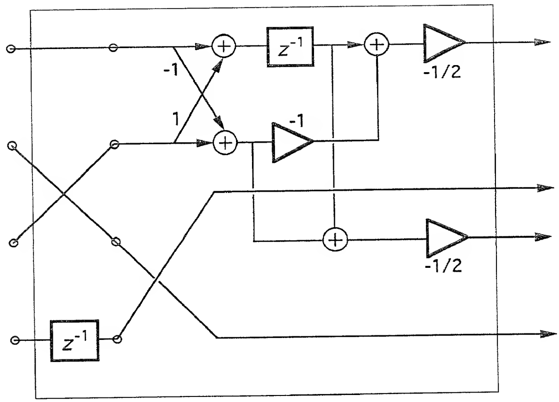


Fig. 14

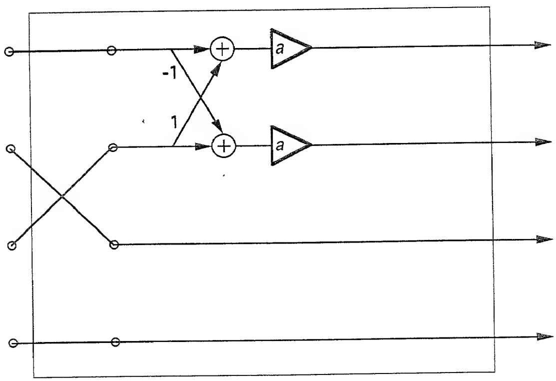


Fig. 15

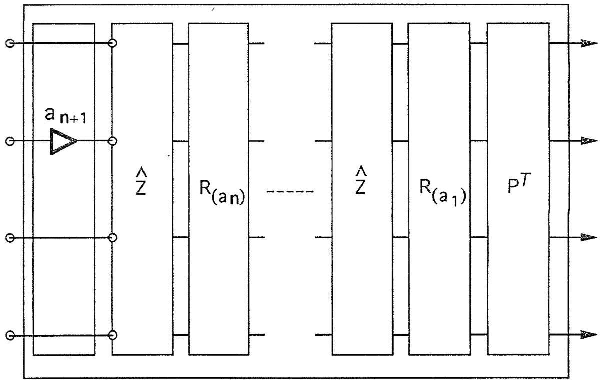


Fig. 16

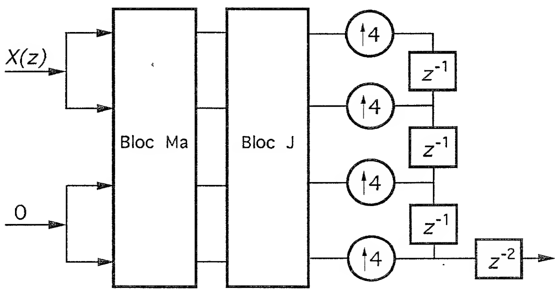


Fig. 17

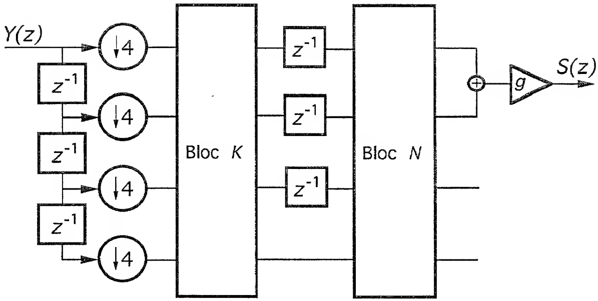


Fig. 18

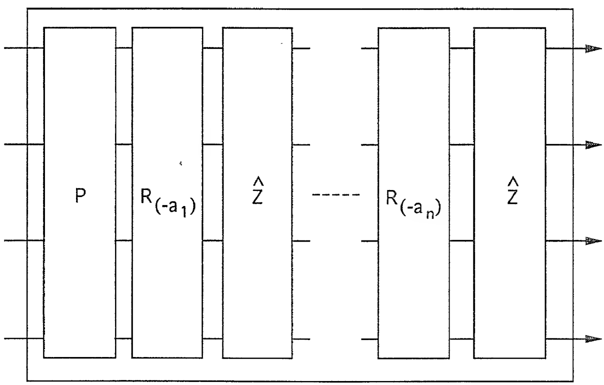
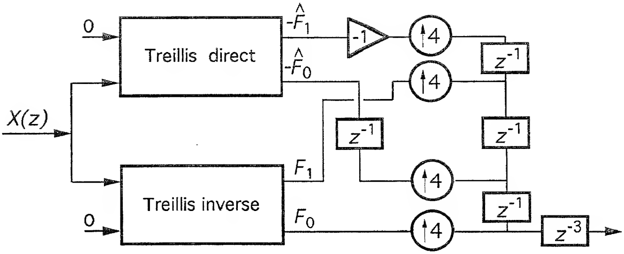


Fig. 19


0
Treillis direct
$-\hat{F}_1$
$-\hat{F}_0$
X(z)
0
Treillis inverse
F_1
F_0
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↑4
↑4
↑4
↑4
z^{-1}
z^{-1}
z^{-1}
z^{-1}
z^{-3}

Fig. 20

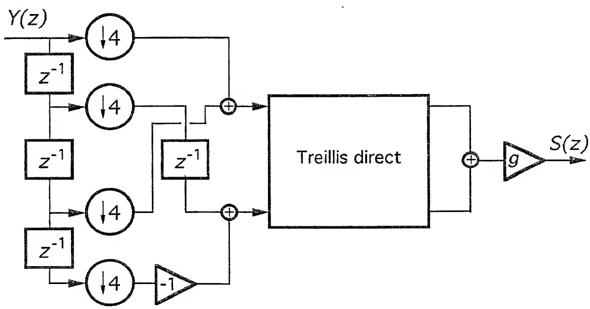
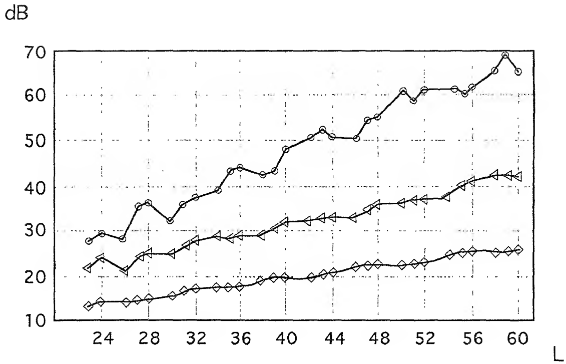

Y(z)
↓4
↓4
↓4
↓4
z^{-1}
z^{-1}
z^{-1}
z^{-1}
-1
Treillis direct
g
S(z)

Fig. 21


dB
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60
50
40
30
20
10
24
28
32
36
40
44
48
52
56
60
L

Fig. 22

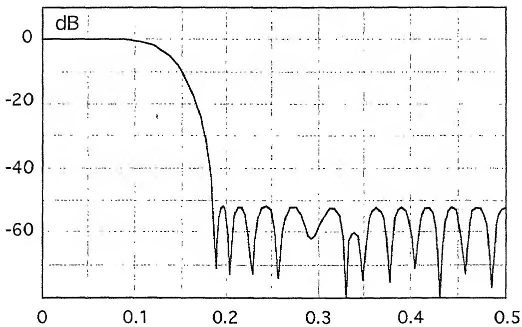
dB
0
-20
-40
-60
0
0.1
0.2
0.3
0.4
0.5

Fig. 23

Attorney Docket No. 9320.114USWO

MERCHANT & GOULD P.C.

United States Patent Application

COMBINED DECLARATION AND POWER OF ATTORNEY.

As a below named inventor I hereby declare that: my residence, post office address and citizenship are as stated below next to my name; that

I verily believe I am the original, first and sole inventor (if only one name is listed below) or a joint inventor (if plural inventors are named below) of the subject matter which is claimed and for which a patent is sought on the invention entitled: METHOD FOR THE MAKING OF DIGITAL NYQUIST FILTERS WITH NULL INTERSYMBOL INTERFERENCE AND CORRESPONDING FILTERING DEVICE

The specification of which
a. ☐ is attached hereto
b. ☒ was filed on as application serial no. and was amended on (if applicable) (in the case of a PCT-filed application) described and claimed in international no. PCT/FR99/01902 filed July 30, 1999 and as amended on (if any), which I have reviewed and for which I solicit a United States patent.

I hereby state that I have reviewed and understand the contents of the above-identified specification, including the claims, as amended by any amendment referred to above.

I hereby claim foreign priority benefits under Title 35, United States Code, § 119/365 of any foreign application(s) for patent or inventor's certificate listed below and have also identified below any foreign application for patent or inventor's certificate having a filing date before that of the application on the basis of which priority is claimed:

a. ☐ no such applications have been filed.
b. ☒ such applications have been filed as follows:

FOREIGN APPLICATION(S), IF ANY, CLAIMING PRIORITY UNDER 35 USC § 119			
COUNTRY	**APPLICATION NUMBER**	**DATE OF FILING (day, month, year)**	**DATE OF ISSUE (day, month, year)**
France	98 09958	30 July 1998	
ALL FOREIGN APPLICATION(S), IF ANY, FILED BEFORE THE PRIORITY APPLICATION(S)			
COUNTRY	**APPLICATION NUMBER**	**DATE OF FILING (day, month, year)**	**DATE OF ISSUE (day, month, year)**

I hereby claim the benefit under Title 35, United States Code, § 120/365 of any United States and PCT international application(s) listed below and, insofar as the subject matter of each of the claims of this application is not disclosed in the prior United States application in the manner provided by the first paragraph of Title 35, United States Code, § 112, I acknowledge the duty to disclose material information as defined in Title 37, Code of Federal Regulations, § 1.56(a) which occurred between the filing date of the prior application and the national or PCT international filing date of this application.

U.S. APPLICATION NUMBER	DATE OF FILING (day, month, year)	STATUS (patented, pending, abandoned)

I hereby claim the benefit under Title 35, United States Code § 119(e) of any United States provisional application(s) listed below:

U.S. PROVISIONAL APPLICATION NUMBER	DATE OF FILING (Day, Month, Year)

I acknowledge the duty to disclose information that is material to the patentability of this application in accordance with Title 37, Code of Federal Regulations, § 1.56 (reprinted below):

§ 1.56 Duty to disclose information material to patentability.

(a) A patent by its very nature is affected with a public interest. The public interest is best served, and the most effective patent examination occurs when, at the time an application is being examined, the Office is aware of and evaluates the teachings of all information material to patentability. Each individual associated with the filing and prosecution of a patent application has a duty of candor and good faith in dealing with the Office, which includes a duty to disclose to the Office all information known to that individual to be material to patentability as defined in this section. The duty to disclose information exists with respect to each pending claim until the claim is canceled or withdrawn from consideration, or the application becomes abandoned. Information material to the patentability of a claim that is canceled or withdrawn from consideration need not be submitted if the information is not material to the patentability of any claim remaining under consideration in the application. There is no duty to submit information which is not material to the patentability of any existing claim. The duty to disclose all information known to be material to patentability is deemed to be satisfied if all information known to be material to patentability of any claim issued in a patent was cited by the Office or submitted to the Office in the manner prescribed by §§ 1.97(b)-(d) and 1.98. However, no patent will be granted on an application in connection with which fraud on the Office was practiced or attempted or the duty of disclosure was violated through bad faith or intentional misconduct. The Office encourages applicants to carefully examine:

(1) prior art cited in search reports of a foreign patent office in a counterpart application, and

(2) the closest information over which individuals associated with the filing or prosecution of a patent application believe any pending claim patentably defines, to make sure that any material information contained therein is disclosed to the Office.

(b) Under this section, information is material to patentability when it is not cumulative to information already of record or being made of record in the application, and

(1) It establishes, by itself or in combination with other information, a prima facie case of unpatentability of a claim; or

(2) It refutes, or is inconsistent with, a position the applicant takes in:

(i) Opposing an argument of unpatentability relied on by the Office, or

(ii) Asserting an argument of patentability.

A prima facie case of unpatentability is established when the information compels a conclusion that a claim is unpatentable under the preponderance of evidence, burden-of-proof standard, giving each term in the claim its broadest reasonable construction consistent with the specification, and before any consideration is given to evidence which may be submitted in an attempt to establish a contrary conclusion of patentability.

(c) Individuals associated with the filing or prosecution of a patent application within the meaning of this section are:

(1) Each inventor named in the application:

(2) Each attorney or agent who prepares or prosecutes the application; and

(3) Every other person who is substantively involved in the preparation or prosecution of the application and who is associated with the inventor, with the assignee or with anyone to whom there is an obligation to assign the application.

(d) Individuals other than the attorney, agent or inventor may comply with this section by disclosing information to the attorney, agent, or inventor.

(e) In any continuation-in-part application, the duty under this section includes the duty to disclose to the Office all information known to the person to be material to patentability, as defined in paragraph (b) of this section, which became available between the filing date of the prior application and the national or PCT international filing date of the continuation-in-part application.

I hereby appoint the following attorney(s) and/or patent agent(s) to prosecute this application and to transact all business in the Patent and Trademark Office connected herewith:

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Xu, Min S.	Reg. No. 39,536
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Please direct all correspondence in this case to Merchant & Gould P.C. at the address indicated below:

Merchant & Gould P.C.
P.O. Box 2903
Minneapolis, MN 55402-0903

23552
PATENT TRADEMARK OFFICE

I hereby declare that all statements made herein of my own knowledge are true and that all statements made on information and belief are believed to be true; and further that these statements were made with the knowledge that willful false statements and the like so made are punishable by fine or imprisonment, or both, under Section 1001 of Title 18 of the United States Code and that such willful false statements may jeopardize the validity of the application or any patent issued thereon.

2	Full Name Of Inventor	Family Name Siohan	First Given Name Pierre	Second Given Name
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Signature of Inventor 201:		P. Lo	Date:	1/02/01
2	Full Name Of Inventor	Family Name Pinchon	First Given Name Didier	Second Given Name
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Signature of Inventor 202:		D. Pinchon	Date:	6 Feb, 2001